Int. J. Heat Mass Transfer. Vol. 35, No. 11, pp. 3041–3046, 1992
Printed in Great Britain

0017–9310/92 $5.00+0.00

Non-Darcian effects on mixed convection about a vertical cylinder embedded in a saturated porous medium

CHA'O-KUANG CHEN and CHIEN-HSIN CHEN
Department of Mechanical Engineering, National Cheng Kung University, Tainan, Taiwan, Republic of China

and

W. J. MINKOWYCZ and U. S. GILL†
Department of Mechanical Engineering, University of Illinois at Chicago, Box 4348, Chicago, IL 60680, U.S.A.

(*Received* 18 *May* 1990 *and in final form* 21 *January* 1991)

Abstract—The problem of mixed convection flow over a vertical cylinder in fluid-saturated porous media has been analyzed by taking the non-Darcian effects into consideration. These effects include the no-slip boundary condition, flow inertial force, variable porosity, and transverse thermal dispersion. The numerical solution of the governing equations has been obtained to demonstrate the important influence of these non-Darcian flow phenomena on fluid flow and heat transfer. The results show that thermal dispersion tends to enhance the heat transfer rate, while boundary and inertia effects decrease it.

INTRODUCTION

CONVECTIVE heat transfer and fluid flow in porous media has recently received considerable attention in connection with geophysical and engineering applications. Such applications include geothermal systems, chemical catalytic reactors, packed-sphere beds, grain storage and thermal insulation engineering. The majority of the existing studies pertinent to flow through porous media are for the case of a Darcy flow. Darcy's law, however, is found to be inadequate in flow situations when there is a solid boundary and the Reynolds number based on the pore size is greater than unity. It is, therefore, necessary to include the boundary and inertia effects in the momentum equations. These effects have been investigated for convective flow and heat transfer about a flat plate embedded in porous media [1–3].

The problem of mixed convection flow over a thin vertical cylinder embedded in a saturated porous medium was studied by Kumari and Nath [4] using a non-Darcy model to account for the inertia effects. Other non-Darcian flow effects which may be important on the same problem have not however, been considered. The aim of the present work is to examine the various non-Darcian effects on the mixed convection about an isothermal vertical cylinder immersed in a fluid-saturated porous medium. Boundary effects can be modeled by adding a viscous-stress term to the momentum equation. A velocity squared term is incorporated into the momentum equation to account for the inertia effects. When the effect of porosity is considered, the flow channeling will occur adjacent to the surface [6, 7]. This effect is demonstrated to be very important on heat transfer. In studying the non-uniform porosity effects, a simple exponential function will be used to approximate the porosity variation in the vicinity of the solid boundary. As pointed out by Cheng [8] and Plumb [9], transverse thermal dispersion effects may become significant when inertia effects are prevalent. This thermal dispersion effect is also examined in the present study. As will be shown in the following sections, the above mentioned non-Darcian effects significantly alter the flow and heat transfer characteristics from those predicted by the Darcy flow model.

ANALYSIS

The steady mixed convection flow over an isothermal vertical cylinder of radius r_0 embedded in a saturated porous medium is considered. It is assumed that the convective fluid and the porous matrix are in local thermodynamic equilibrium, and that the Boussinesq approximation is valid. The boundary-layer equations in a cylindrical coordinate system are given as [2]

$$\frac{\partial}{\partial x}(ru)+\frac{\partial}{\partial r}(rv) = 0 \tag{1}$$

$$\frac{\mu}{K}u+\rho Cu^2 = -\frac{\partial p}{\partial x}+\frac{\mu}{\phi}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\rho g\beta(T-T_\infty) \tag{2}$$

† Present address: National Aeronautics and Space Administration, Marshall Space Flight Center, Huntsville, AL 35812, U.S.A.

NOMENCLATURE

A	empirical constant in equation (6)
B	constant defined in equation (16)
C	inertia coefficient
c	specific heat of the fluid
Da	Darcy number, K_∞/x^2
D_t	empirical constant defined in equation (17)
d	particle diameter
f	dimensionless stream function
Gr	Grashof number, $g\beta K_\infty x(T_w - T_\infty)/v^2$
g	gravitational constant
h	heat transfer coefficient
K	permeability
k_d	stagnant thermal conductivity
k_e	effective thermal conductivity
k_f	thermal conductivity of fluid
k_s	thermal conductivity of particles
k_t	thermal dispersion conductivity
N	empirical constant in equation (6)
Nu	Nusselt number, hx/k_e
p	pressure
Pe_d	Peclet number based on particle diameter, $u_\infty d/\alpha_f$
Pe	Peclet number, $u_\infty x/\alpha_f$
Pr	Prandtl number of the fluid
q	local heat flux
Re	Reynolds number, $u_\infty x/v$
r	radial coordinate
r_0	radius of cylinder
T	temperature
u	x-component velocity
u_∞	convective velocity, $-(K_\infty/\mu)(dp/dx)$
v	r-component velocity
x	axial coordinate.

Greek symbols

α_e	effective thermal diffusivity
α_f	thermal diffusivity of fluid
β	thermal expansion coefficient
Γ	parameter in equation (14)
η	pseudo-similarity variable
θ	dimensionless temperature
λ	thermal conductivity ratio of the solid phase to fluid phase
μ	viscosity of the fluid
v	kinematic viscosity of fluid
ξ	dimensionless streamwise coordinate
ρ	density of the fluid
σ	α_e/α_f
ϕ	porosity
ψ	stream function.

Subscripts

∞	quantities away from the wall
w	quantities at wall.

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\frac{1}{r}\frac{\partial}{\partial r}\left(a_e r\frac{\partial T}{\partial r}\right) \tag{3}$$

where u and v are the components of velocity in the x- and r-directions; T, p, and g are the temperature, pressure, and gravitational constant; ρ, μ, and β are the density, viscosity, and the thermal expansion coefficient of the fluid; K, C, and ϕ are the permeability, inertia coefficient, and porosity of the porous medium; and $a_e = k_e/\rho c$ is the effective thermal diffusivity of the porous medium with k_e denoting the effective thermal conductivity of the saturated porous medium and ρc the product of the density and specific heat of the fluid. The appropriate boundary conditions for this problem are

$$u = v = 0,\quad T = T_w \quad \text{at} \quad r = r_0 \tag{4}$$

$$u = u_\infty,\quad T = T_\infty \quad \text{as} \quad r \to \infty. \tag{5}$$

In order to study the variable porosity effects, an exponential decrease is usually assumed to approximate the near-wall porosity variation such as in packed-sphere beds [10]

$$\phi = \phi_\infty + \{1 + A\exp[-N(r-r_0)/d]\} \tag{6}$$

where $\phi_\infty = 0.4$ is the free-stream porosity, d the particle diameter, and A and N are the empirical constants which depend on the packing of particles near the solid wall. The value for A is determined so that the porosity at the wall is 0.9. The value of $N = 6$ is used to represent the decay of porosity from the solid wall [10]. Both the permeability K and the inertia coefficient C of the porous matrix can be expressed in terms of the particle diameter and porosity from the correlations developed by Ergun [11]

$$K = \frac{d^2\phi^3}{150(1-\phi)^2} \tag{7}$$

$$C = \frac{1.75(1-\phi)}{d\phi^3}. \tag{8}$$

The continuity equation, equation (1), is satisfied identically by introducing the stream function ψ as

$$ru = \frac{\partial\psi}{\partial r} \quad \text{and} \quad rv = -\frac{\partial\psi}{\partial x}. \tag{9}$$

Equations (2) and (3) along with the boundary conditions (4) and (5) can be non-dimensionalized by defining the following transformations

$$\eta = \frac{r^2 - r_0^2}{2r_0}\left(\frac{u_\infty}{\alpha_f x}\right)^{1/2},\quad \xi = \frac{2}{r_0}(\alpha_f x/u_\infty)^{1/2}$$

$$f(\xi,\eta) = \psi/[r_0(\alpha_f u_\infty x)^{1/2}],\quad \theta = \frac{T - T_\infty}{T_w - T_\infty}. \tag{10}$$

The governing equations (2) and (3) in terms of these new variables are

$$\frac{Da\,Pe}{\phi}\frac{\partial}{\partial\eta}\left((1+\xi\eta)\frac{\partial^2 f}{\partial\eta^2}\right)-\Gamma\left(\frac{C}{C_\infty}\right)\left(\frac{\partial f}{\partial\eta}\right)^2 - \left(\frac{K_\infty}{K}\right)\frac{\partial f}{\partial\eta}+\frac{Gr}{Re}\theta+1=0 \quad (11)$$

$$\sigma\frac{\partial}{\partial\eta}\left((1+\xi\eta)\frac{\partial\theta}{\partial\eta}\right)+\left(\frac{1}{2}f+(1+\xi\eta)\frac{\partial\sigma}{\partial\eta}\right)\frac{\partial\theta}{\partial\eta} = \frac{1}{2}\xi\left(\frac{\partial f}{\partial\eta}\frac{\partial\theta}{\partial\xi}-\frac{\partial\theta}{\partial\eta}\frac{\partial f}{\partial\xi}\right) \quad (12)$$

where $Da = K_\infty/x^2$, $Pe = u_\infty x/\alpha_f$, $\Gamma = K_\infty C_\infty u_\infty/\nu$, $\sigma = \alpha_e/\alpha_f$ and $Gr/Re = g\beta K_\infty(T_w-T_\infty)/u_\infty\nu$. Boundary conditions (4) and (5) reduce to

$$f'=0,\quad f+\xi\frac{\partial f}{\partial\xi}=0,\quad \theta=1 \quad \text{at} \quad \eta=0 \quad (13)$$

$$f'=\frac{-1+\sqrt{(4\Gamma+1)}}{2\Gamma},\quad \theta=0 \quad \text{as}\ \eta\to\infty$$

$$C=C_\infty,\quad K=K_\infty \qquad \text{as}\ \eta\to\infty \quad (14)$$

where the primes indicate differentiation with respect to η. The free-stream boundary condition on the velocity is obtained from the momentum equation, equation (14), by neglecting the viscous and buoyancy terms.

It is well known that the effective thermal conductivity k_e of a saturated porous medium is composed of a sum of the stagnant thermal conductivity k_d (due to molecular diffusion) and the thermal dispersion conductivity k_t (due to mechanical dissipation), i.e.

$$k_e = k_d + k_t. \quad (15)$$

The stagnant thermal conductivity for packed-sphere beds can be given by the following semi-analytical expression [12]

$$\frac{k_d}{k_f} = [1-\sqrt{(1-\phi)}]+\frac{2\sqrt{(1-\phi)}}{1-\lambda B} \times\left[\frac{(1-\lambda)B}{(1-\lambda B)^2}\ln\left(\frac{1}{\lambda B}\right)-\frac{B+1}{2}-\frac{B-1}{1-\lambda B}\right] \quad (16)$$

where $B = 1.25[(1-\phi)/\phi]^{10/9}$ and $\lambda = k_f/k_s$ is the ratio of the thermal conductivity of fluid to that of particles. Equation (10) reveals that the stagnant thermal conductivity is a function of position for variable porosity media. As proposed by Hsu and Cheng [13], the thermal dispersion conductivity can be expressed in terms of the new variables as

$$\frac{k_t}{k_f} = D_t\frac{1-\phi}{\phi^2}Pe_d f' \quad (17)$$

where D_t is an empirical constant space and $Pe_d = u_c d/\alpha_f$.

The most important result to be given is the heat transfer along the vertical cylinder. Consider first the local surface heat flux which can be written as

$$q = -k_e\left(\frac{\partial T}{\partial r}\right)_{r=r_0}. \quad (18)$$

The local heat transfer rate at the wall can be presented conveniently in terms of the local Nusselt number Nu, which is defined as $Nu = hx/k_e$, where h is the local heat transfer coefficient. Combining equation (18) with the definition of h, i.e. $q = h(T_w-T_\infty)$, results in

$$Nu/Pe^{1/2} = -\theta'(\xi,0) \quad (19)$$

which is recognized as the local heat transfer parameter.

RESULTS AND DISCUSSION

Numerical solutions to equations (14) and (15) for various flow models were obtained using the Keller Box method which is described in great detail in ref. [14]. In carrying out the numerical simulations, the following values of physical quantities were employed: $Pr = 3.54$, $u_c = 0.01$ m s^{-1}, and $k_s = 1.05$ W m^{-1} K^{-1} for glass beads of diameters 3 mm and 5 mm. Since various non-Darcian effects are taken into consideration, the following symbols are used: nBnIU, which indicates no Boundary, no Inertia, and Uniform porosity effects; BIV, which refers to Boundary, Inertia, and Variable porosity effects, etc. It may be remarked that nBnIU is the Darcy flow case and nBIU is the case reported in ref. [4]. Also nBnIU with $\xi = 0$ reduces to the case of mixed convection about a vertical flat plate studied by Cheng [5]. In order to confirm the accuracy of the present numerical method, the results of these special cases have been compared and found in good agreement with those of refs. [4, 5]. The comparison is not presented here for brevity.

Figures 1–8 present the results with dispersion effects neglected. Numerical results for the local heat transfer parameter $Nu/\sqrt{(Pe)}$ are illustrated in Figs.

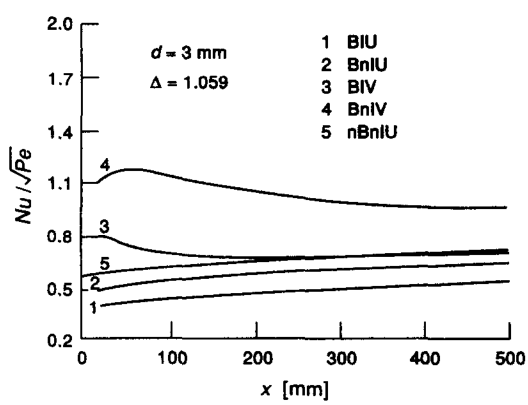

FIG. 1. Local heat transfer parameters for forced convection ($Gr/Re = 0$).

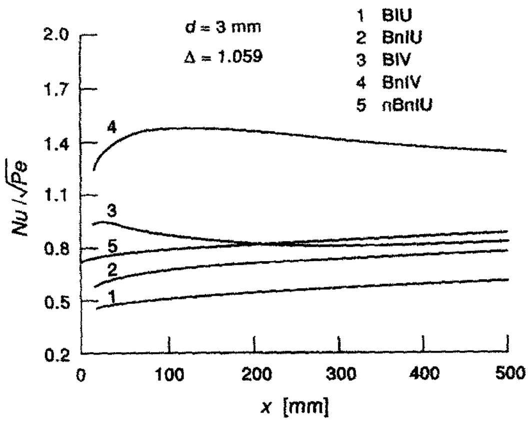

FIG. 2. Local heat transfer parameters for forced convection ($Gr/Re = 1$).

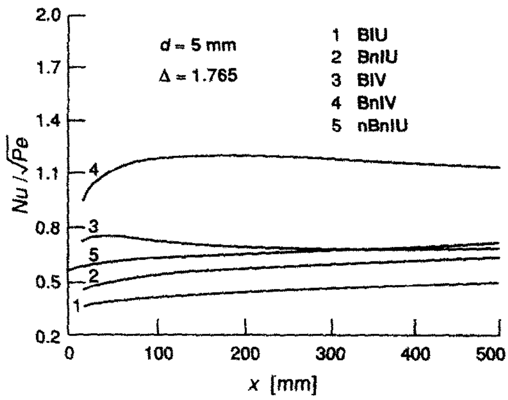

FIG. 3. Local heat transfer parameters for forced convection ($Gr/Re = 0$).

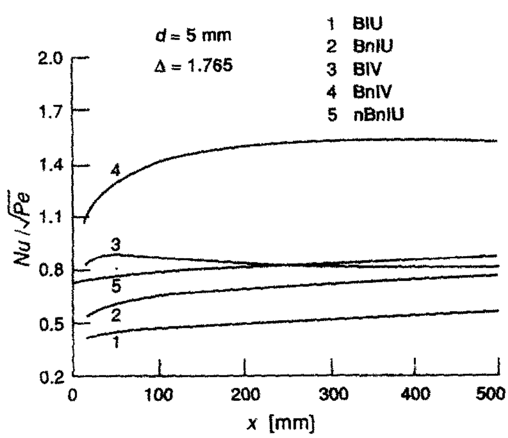

FIG. 4. Local heat transfer parameters for forced convection ($Gr/Re = 1$).

1–4. Results for forced convection ($Gr/Re = 0$) are also shown to indicate the fact that buoyancy force augments the heat transfer rates. The no-slip boundary effect, which is governed by the parameter *Da Pe*, reduces the heat transfer as compared to the case of a Darcy flow. The decrease in the heat transfer

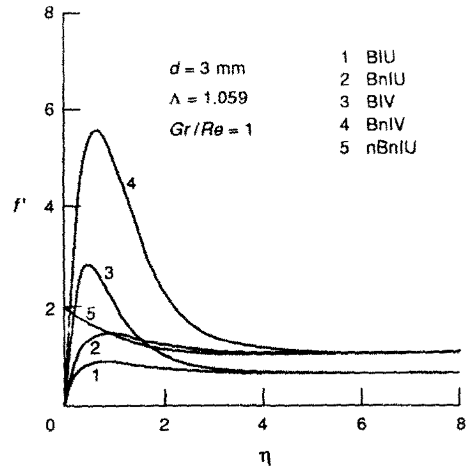

FIG. 5. The velocity distribution at $x = 20$ mm.

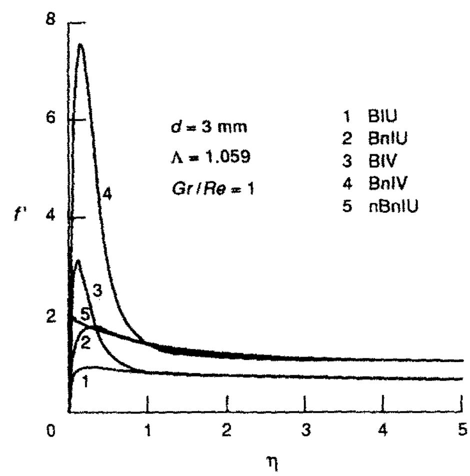

FIG. 6. The velocity distributions at $x = 350$ mm.

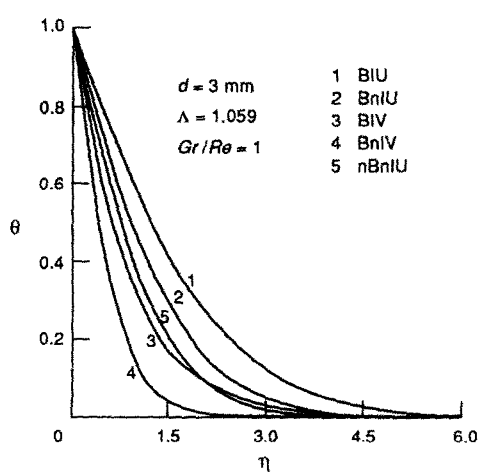

FIG. 7. The temperature distributions at $x = 20$ mm.

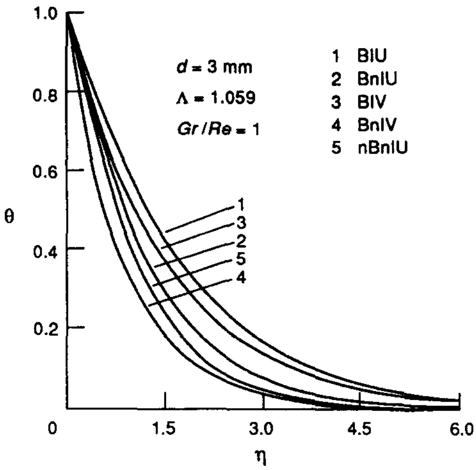

FIG. 8. The temperature distributions at $x = 350$ mm.

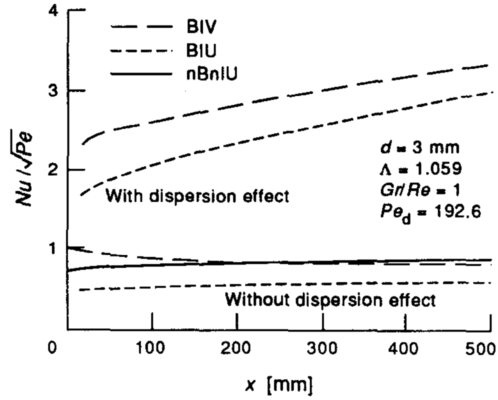

FIG. 9. Influence of thermal dispersion on the local heat transfer parameters.

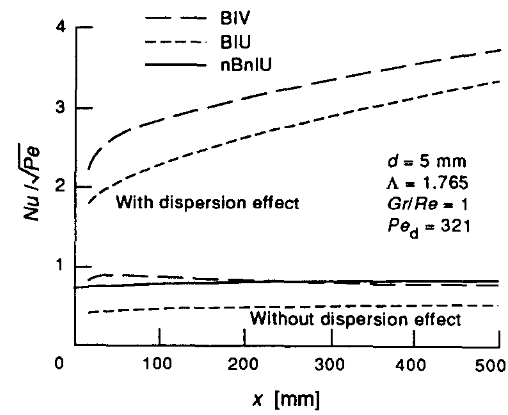

FIG. 10. Influence of thermal dispersion on the local heat transfer parameters.

parameter is more significant near the leading edge, as shown in the figures, which imply that the boundary effect is more important near the leading edge. Like the boundary effect, the effect of flow inertia is to decrease the heat transfer rate. This is similar to what has been reported in ref. [2] for forced convection over a flat plate. The importance of the inertia effect is characterized by the parameter Γ defined in equation (11). This parameter is seen to be proportional to the particle diameter d, hence the heat transfer is more reduced for porous media with large particles. The numerical solutions also indicate the enhancement in heat transfer due to flow channeling. It should be noticed that the heat transfer will be overestimated without the inclusion of inertia effects in non-uniform porous media. When the boundary, inertia, and variable porosity effects are considered simultaneously, the heat transfer can be either enhanced or reduced, as compared to the Darcy case, depending on the balance between these mechanisms.

Typical velocity distributions with various non-Darcian effects are plotted in Fig. 5 for a position near the leading edge and in Fig. 6 for a downstream position. It can be concluded that the boundary and inertia effects tend to decrease the velocity, while the variable porosity effect causes an overshoot of velocity near the wall. This channeling profile is created by the non-uniform porosity distribution with a high porosity region near the wall. At downstream locations the velocity overshooting is seen to be confined in a narrower region than that near the leading edge. Also from these figures it is noted that the Darcy flow model (nBnIU) allows a slip velocity on the solid boundary. The free-stream velocity for the case with inertia effect included is given by equation (14). Figures 7 and 8 present the dimensionless temperature profiles corresponding to the velocity profiles presented in Figs. 5 and 6. As seen in the figures, both boundary and inertia effects lead to a thicker thermal boundary layer but a smaller temperature gradient at the wall.

The effect of thermal dispersion on the heat transfer parameter can be observed in Figs. 9 and 10. This effect is expected to become important in the case where inertia effects are prevalent. The empirical constant D_t defined in equation (17) should be determined from experiment. In the present study a fixed value of 0.02 [13] is used to examine qualitatively the dispersion effect. The results show that heat transfer is increased greatly by taking into consideration the thermal dispersion effect. This large enhancement in heat transfer caused by the dispersive transport can be attributed to the better mixing of convective fluid within the pores. As verified from the figures, thermal dispersion has a more pronounced effect at a higher value of Pe_d.

CONCLUSION

The analysis is performed to investigate the significance of non-Darcian flow effects on mixed convection from a heated vertical cylinder embedded in fluid-saturated porous media. It is seen that the velocity and temperature profiles, predicted by including the non-Darcian effects, differ significantly from those based on Darcy's law. When both the boundary and inertial effects are incorporated in the analysis, the velocity in the boundary layer is reduced, resulting in a lower heat transfer rate. The flow channeling effect, which is due to the variation in porosity near the wall, enhances the momentum transport in the boundary layer and results in an increase in heat transfer. Whether heat transfer is enhanced or reduced depends on the relative magnitude of these effects. Also, the results demonstrate that dispersion dramatically increases the thermal communication between the porous matrix and the solid boundary.

REFERENCES

1. O. A. Plumb and J. C. Huenefeld, Non-Darcy natural convection from heated surfaces in saturated porous media, *Int. J. Heat Mass Transfer* **24**, 765–768 (1981).
2. K. Vafai and C. L. Tien, Boundary and inertia effects on flow and heat transfer in porous media, *Int. J. Heat Mass Transfer* **24**, 195–203 (1981).
3. C. T. Hsu and P. Cheng, The Brinkman model for natural convection about a semi-infinite flat plate in a porous medium, *Int. J. Heat Mass Transfer* **28**, 683–697 (1985).
4. M. Kumari and G. Nath, Non-Darcy mixed convection boundary layer flow on a vertical cylinder in a saturated porous medium, *Int. J. Heat Mass Transfer* **32**, 183–187 (1989).
5. P. Cheng, Combined free and forced convection flow about inclined surfaces in porous media, *Int. J. Heat Mass Transfer* **20**, 807–814 (1977).
6. D. Vortmeyer and J. Schuster, Evaluation of steady flow profiles in rectangular and circular packed beds, *Chem. Engng Sci.* **38**, 1691–1699 (1983).
7. B. C. Chandrasekhara and P. M. Namboodiri, Influence of variable permeability on combined free and forced convection about inclined surfaces in porous media, *Int. J. Heat Mass Transfer* **28**, 199–206 (1985).
8. P. Cheng, Thermal dispersion effects in non-Darcian convective flows in a saturated porous medium, *Lett. Heat Mass Transfer* **8**, 267–270 (1981).
9. O. A. Plumb, The effect of thermal dispersion on heat transfer in packed bed boundary layers, *Proc. 1st ASME/JSME Thermal Engineering Joint Conference*, Vol. 2, pp. 17–21 (1983).
10. R. F. Benenati and C. B. Brosilow, Void fraction distribution in beds of spheres, *A.I.Ch.E. Jl* **8**, 359–361 (1962).
11. S. Ergun, Fluid flow through packed columns, *Chem. Engng Prog.* **48**, 89–94 (1952).
12. P. Zehner und E. U. Schluender, Waermeleitfahigkeit von schuettungen bei massigen temperaturen, *Chemie-Ingr-Tech.* **42**, 933–941 (1970).
13. C. T. Hsu and P. Cheng, Closure schemes of the macroscopic energy equation for convective heat transfer in porous media, *Int. Commun. Heat Mass Transfer* **15**, 689–703 (1988).
14. T. Cebeci and P. Bradshaw, *Momentum Transfer in Boundary Layers*. Hemisphere, Washington, DC (1977).

EFFETS NON DARCIEN SUR LA CONVECTION MIXTE AUTOUR D'UN CYLINDRE VERTICAL NOYE DANS UN MILIEU POREUX SATURE

Résumé—On considère le problème de la convection mixte autour d'un cylindre vertical dans un milieu poreux en tenant compte des effets non darciens. Ces effets incluent la condition de non glissement à la frontière, la force d'inertie, la porosité variable et la dispersion thermique transverse. La solution numérique des équations est obtenue pour montrer l'influence importante des phénomènes non darciens sur l'écoulement du fluide et le transfert thermique. Les résultats montrent que la dispersion thermique tend à augmenter le flux thermique transféré, tandis que les effets de frontière et d'inertie le diminuent.

NICHT-DARCY'SCHE EINFLÜSSE AUF DIE MISCHKONVEKTION UM EINEN VERTIKALEN ZYLINDER IN EINEM GESÄTTIGTEN PORÖSEN MEDIUM

Zusammenfassung—Das Problem der Mischkonvektion entlang eines senkrechten Zylinders in einem fluidgesättigten porösen Medium wird unter Berücksichtigung nicht-Darcy'scher Effekte analysiert. Diese Einflüsse enthalten die Haftbedingung an der Wand, die Trägheit der Strömung, variable Porosität und quergerichtete thermische Dispersion. Die zugrundeliegenden Gleichungen werden numerisch gelöst, um den wichtigen Einfluß dieser nicht-Darcy'schen Phänomene auf Strömung und Wärmeübergang zu demonstrieren. Die Ergebnisse zeigen, daß thermische Dispersion gewöhnlich den Wärmeübergang intensiviert, während dieser durch Rand- und Trägheitseffekte verringert wird.

ВЛИЯНИЕ НЕДАРСОВЫХ ЭФФЕКТОВ НА СМЕШАННУЮ КОНВЕКЦИЮ У ВЕРТИКАЛЬНОГО ЦИЛИНДРА, ПОМЕЩЕННОГО В НАСЫЩЕННУЮ ПОРИСТУЮ СРЕДУ

Аннотация—С учетом недарсовых эффектов анализируется задача смешанноконвективного обтекания вертикального цилиндра, помещаемого в насыщенные жидкостью пористые среды. Указанные эффекты включают граничное условие отсутствия скольжения, инерционную силу течения, изменяющуюся пористость и поперечное рассеяние тепла. Получено численное решение определяющих уравнений, которое демонстрирует существенное влияние недарсовых явлений на течение жидкости и теплоперенос. Результаты показывают, что рассеяние тепла приводит к интенсификации теплопереноса, в то время как граничные и инерционные эффекты его уменьшают.